# Scenario Generation via Convex Polytopes

***Understanding Cost Tolerance in NATO Common Funded Capability Delivery Programmes***

Dr. Bohdan Kaluzny and LTC Pierre Han

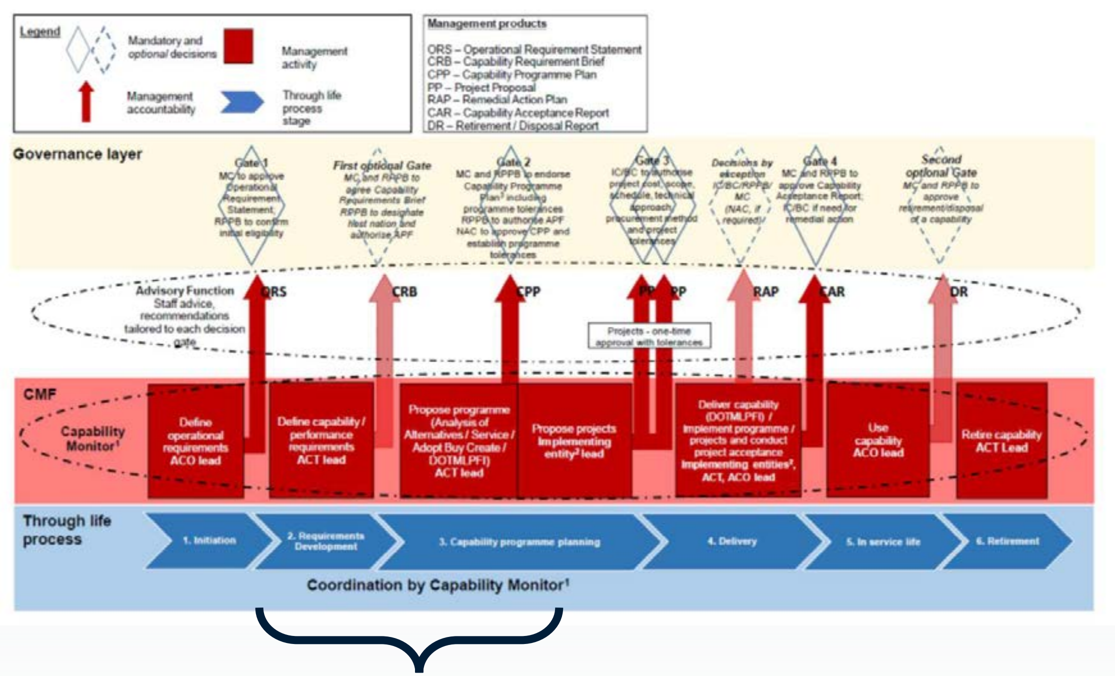
# Outline

- Common Funded Capability Delivery
- Cost Tolerances and Scenarios
- Scenario Generation via Polytope Analysis
- Alternative Approaches to Scenario Generation
- Conclusion

# 01. Common Funded Capability Delivery

# NATO Common Funded Capability Delivery Governance Model (CFCDGM)

- New governance model approved in 2018
- Delivery of the right capabilities on time
  - warfighting capabilities required by NATO Commanders and the NATO Enterprise
- Six process stages
- Separation of Management and Governance
- Governance by Exception




ACT Involvement

# Programme Cost Estimates

- All programmes are required to produce Life Cycle Cost estimates as part of the governance model
- Rough Order of Magnitude in the $3^{rd}$ stage
- Holistic assessment - considers all elements (*DOTMLPFI*)
- NATO Standard ALCPP-1 (Guidance on Life Cycle Costs)

# Governance by Exception

- Environment of trust and accountability
- Management of programmes and projects within agreed tolerances (as opposed to provision of contingency)
- Tolerance: allowable variance in
  - Cost
  - Scope
  - Schedule
  - Performance

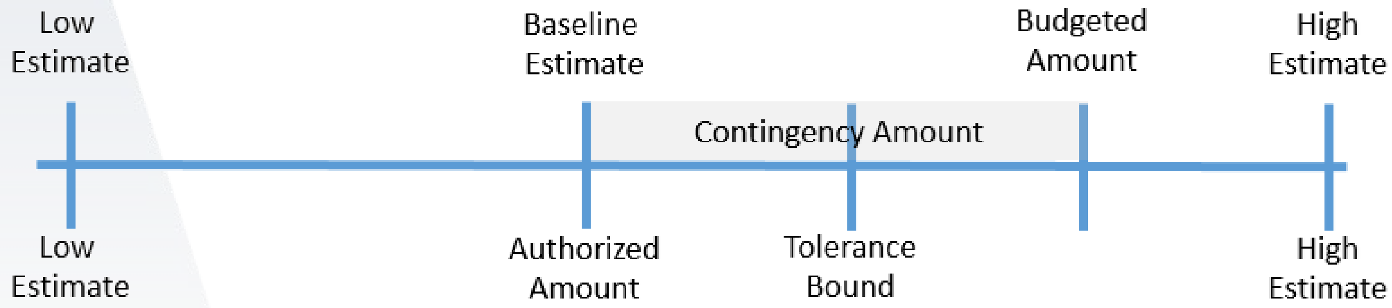
# Cost Tolerance vs. Contingency

## Contingency:

*Costs that will probably occur based on past experience, but with some uncertainty regarding the amount*

## Tolerance in the governance model:

*NATO's degree of risk appetite and the likelihood of change*



# Cost Tolerances in the Governance Model

- Reflect the degree of **risk appetite** and the **likelihood of change**
- Risk management is essential to set the right tolerances
- NATO ACT programme directors are required to specify proposed tolerance levels upon submission of Capability Programme Plans (CPP; or Stage 3)
- …but little to no direction/guidance as to how to determine these…

…**more importantly: what does the tolerance imply?**

# 02. Scenarios and Cost Tolerances

# Cost Tolerance Implications

- What does a **100** Million EUR cost tolerance mean?
  - Under what scenarios will the programme proceed without Governance intervention?
  - Under what conditions (or scenarios) will the programme breach it?

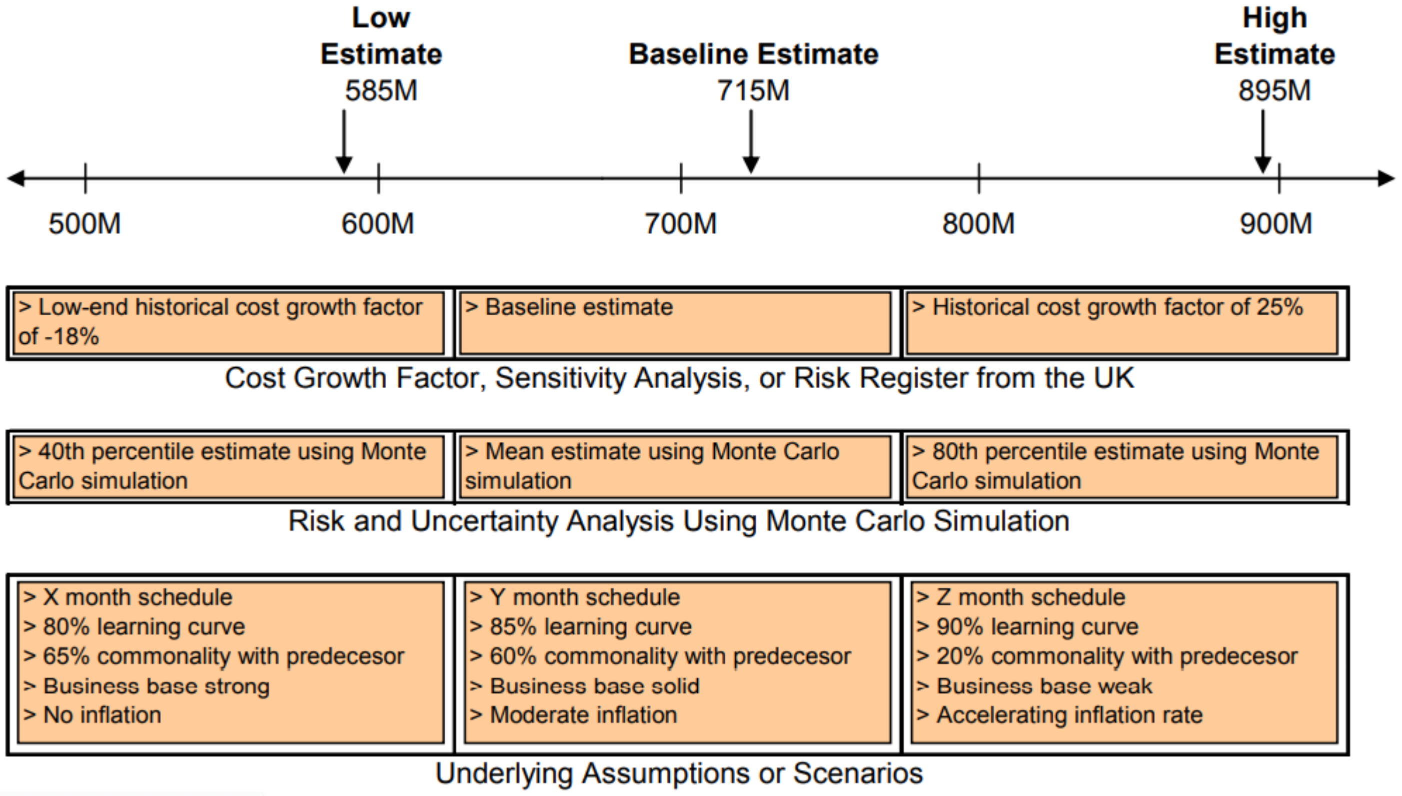
- How to enable and inform Governance and Management to understand the implication?

# What are Scenarios?

- Scenarios are realizable combinations of known-unknowns
- How to generate scenarios?
  - Expert judgement (prone to subjectivity / error)
  - Analysis of comparable, completed programmes (relies on good data collection)
  - Simulation Analysis (low-level of granularity required)
- **Problem Statement:** Given a tolerance threshold, what is a set of scenarios that would be admissible? (alternatively, what scenarios would breach the threshold?)

# Current Best Practice

- NATO TR-SAS-054: Methods and Models for Life Cycle Costing
  - Recommended approach for communicating results of a LCC estimate to senior decision makers
- Simple and intuitive
- Unrealistic blanket assumption across cost drivers for each scenario



03.

# Scenario Generation via Polytope Analysis

# Example: Smart Supply Depot Programme

*[fictional scenario]* NATO is examining options to construct an expansive multi-billion dollar network of Smart Supply Depots. The Programme recommends a $600M tolerance level and Governance would like to understand scenarios which would be covered by the amount.

Expert cost estimation reports reveal four main cost drivers. Various methodologies were employed to derive the financial impacts and likely upper and lower bounds:

| Main Cost Drivers | Cost Driver Unit of Measure | Impact ($Millions/Unit) | Upper /Lower Bounds | Methodology to derive Impact & Bounds |
|---|---|---|---|---|
| Labour rates | 1% pt. of rate | 80 | -10 / +10 | Historical data from infrastructure projects |
| Material | 1% pt. change | 5 | -2 / +5 | Historical data from infrastructure projects |
| Engineering design | 1% pt. of scope | 60 | 0 / 6 | Expert opinion; historical cost data |
| Artificial Intelligence System | 10K lines of code | 1.2 | -100 / +500 | Sensitivity analysis from previous NATO projects |

# Polytope Analysis Approach

- Step 1: Identify main cost risk drivers (e.g., sensitivity analysis)
- Step 2: Quantify impact of individual cost drivers (suitable granularity)
- Step 3: Define lower and upper bounds for each cost driver
- Step 4: Express a system of variables and inequalities
- Step 5: Convert to menu / list description of scenarios

# Example: Smart Supply Depot Programme

- Step 1: Identify main cost risk drivers
  - **Labour rates**
  - **Material**
  - **Engineering design**
  - **Artificial Intelligence Systems (coding requirement)**

# Example: Smart Supply Depot Programme

- Step 2: Quantify impact of individual cost drivers
  - **Labour rates:** 1% pt. change in labour rate results in $80M change
  - **Material:** 1% pt. change to quantity results in $5M change
  - **Engineering design:** 1% pt. change of scope results in $60M change
  - **Artificial Intelligence Systems (code):** 1 unit = ten thousand lines of code; change of one unit results in $1.2M change.

# Example: Smart Supply Depot Programme

- Step 3: Define lower and upper bounds for each cost driver
  - **Labour rates:** as low as -10% points; as high as +10% points
  - **Material:** as low as -2% points; as high as 5% points
  - **Engineering:** as low as 0% points; as high as 6% points
  - **Software:** as low as -100 units less, as high as 500 units more

# Example: Smart Supply Depot Programme

- Steps 4a: Express a system of variables and inequalities
  - $x_1$ = % point deviation in labour rates
  - $x_2$ = % point deviation in material quantity
  - $x_3$ = % point deviation in scope (requirements)
  - $x_4$ = unit change in software (10,000 lines of code)

# Example: Smart Supply Depot Programme

- Step 4b: Express a system of variables and inequalities

| | |
|---|---|
| $-10 \leq x_1 \leq 10$ | Labour rates: as low as -10% points; as high as +10% points |
| $-2 \leq x_2 \leq 5$ | Material: as low as -2% points; as high as 5% points |
| $0 \leq x_3 \leq 6$ | Engineering: as low as 0% points; as high as 6% points |
| $-100 \leq x_4 \leq 500$ | Software: as low as -100 units less, as high as 500 units more |
| $80\ x_1 + 5\ x_2 + 60\ x_3 + 1.2\ x_4 \leq 600$ | **Tolerance threshold: scenarios <u>within</u> limit** |

# Example: Smart Supply Depot Programme

- Step 4b: Express a system of variables and inequalities

| | |
|---|---|
| $-10 \leq x_1 \leq 10$ | Labour rates: as low as -10% points; as high as +10% points |
| $-2 \leq x_2 \leq 5$ | Material: as low as -2% points; as high as 5% points |
| $0 \leq x_3 \leq 6$ | Engineering: as low as 0% points; as high as 6% points |
| $-100 \leq x_4 \leq 500$ | Software: as low as -100 units less, as high as 500 units more |
| ~~$80\ x_1 + 5\ x_2 + 60\ x_3 + 1.2\ x_4 \leq 600$~~ | Tolerance threshold |
| $600 \leq 80\ x_1 + 5\ x_2 + 60\ x_3 + 1.2\ x_4 \leq 600 + \Delta$ | **Tolerance threshold: scenarios <u>at and above</u> the limit (by** $\Delta$) |

# Example: Smart Supply Depot Programme

- Step 5: Convert to menu / list description of scenarios

$$-10 \le x_1 \le 10$$

$$-2 \le x_2 \le 5$$

$$0 \le x_3 \le 6$$

$$-100 \le x_4 \le 500$$

$$80\ x_1 + 5\ x_2 + 60\ x_3 + 1.2\ x_4 \le 600$$

Not useful

# Example: Smart Supply Depot Programme

- Step 5: Convert to menu / list description of scenarios

$$-10 \leq x_1 \leq 10$$

$$-2 \leq x_2 \leq 5$$

$$0 \leq x_3 \leq 6$$

$$-100 \leq x_4 \leq 500$$

$$80\,x_1 + 5\,x_2 + 60\,x_3 + 1.2\,x_4 \leq 600$$

Not useful

List of scenarios

| | Labour | Material | Scope | Software | COST |
|---|---|---|---|---|---|
| a | 1.6 | -2 | 0 | -100 | 0 |
| b | -2.9 | -2 | 6 | -100 | 0 |
| c | -10.0 | -2 | 6 | 375 | 0 |
| d | -10.0 | -2 | 4 | 500 | 0 |
| e | -10.0 | -2 | 6 | 500 | 150 |
| f | -4.4 | -2 | 6 | 500 | 600 |
| g | -10.0 | 5 | 6 | 500 | 185 |
| h | -4.8 | 5 | 6 | 500 | 600 |
| i | -10.0 | 5 | 3 | 500 | 0 |
| j | 4.6 | -2 | 6 | -100 | 600 |
| k | -3.3 | 5 | 6 | -100 | 0 |
| l | -10.0 | 5 | 6 | 346 | 0 |
| m | 4.2 | 5 | 6 | -100 | 600 |
| n | -7.4 | -2 | 0 | 500 | 0 |
| o | 0.1 | -2 | 0 | 500 | 600 |
| p | -7.8 | 5 | 0 | 500 | 0 |
| q | -0.3 | 5 | 0 | 500 | 600 |
| r | 9.1 | -2 | 0 | -100 | 600 |
| s | 1.2 | 5 | 0 | -100 | 0 |

Useful?

# Example: Smart Supply Depot Programme

- Step 5: Convert to menu / list description of scenarios

List of scenarios

| | Labour | Material | Scope | Software | COST |
|---|---|---|---|---|---|
| a | 1.6 | -2 | 0 | -100 | 0 |
| b | -2.9 | -2 | 6 | -100 | 0 |
| c | -10.0 | -2 | 6 | 375 | 0 |
| d | -10.0 | -2 | 4 | 500 | 0 |
| e | -10.0 | -2 | 6 | 500 | 150 |
| f | -4.4 | -2 | 6 | 500 | 600 |
| g | -10.0 | 5 | 6 | 500 | 185 |
| h | -4.8 | 5 | 6 | 500 | 600 |
| i | -10.0 | 5 | 3 | 500 | 0 |
| j | 4.6 | -2 | 6 | -100 | 600 |
| k | -3.3 | 5 | 6 | -100 | 0 |
| l | -10.0 | 5 | 6 | 346 | 0 |
| m | 4.2 | 5 | 6 | -100 | 600 |
| n | -7.4 | -2 | 0 | 500 | 0 |
| o | 0.1 | -2 | 0 | 500 | 600 |
| p | -7.8 | 5 | 0 | 500 | 0 |
| q | -0.3 | 5 | 0 | 500 | 600 |
| r | 9.1 | -2 | 0 | -100 | 600 |
| s | 1.2 | 5 | 0 | -100 | 0 |

Scenario f:

- Favourable labour rates (-4.4%)
- Favourable Material use (-2%)
- Change in scope (6%)
- Increase in software required (500 units)

**Result: At tolerance threshold**

# Algorithms/software implementing the theorem

- Lexicographic Reverse Search (lrs)
- Double Description Method (cdd)
- Primal-Dual algorithm (pd)
- Polymake
- Porta

# 04. Alternative approaches

# Alternative Approach: Morphological Analysis

- Morphological Analysis: Exploration of possible solutions to a multi-dimensional problem
- Enumerate a sufficient number of possible outcomes for each cost driver

| Main Cost Drivers | Cost Driver Unit of Measure | Impact ($Millions) | Upper /Lower Bounds |
|---|---|---|---|
| Labour rates | 1% pt. of rate | 80 | -10 / +10 |
| Material | 1% pt. change | 5 | -2 / +5 |
| Engineering design | 1% pt. of scope | 60 | 0 / 6 |
| Artificial Intelligence System | 10K lines of code | 1.2 | -100 / +500 |

| Labour rates | Material | Engineering design | Artificial Intelligence System |
|---|---|---|---|
| -10%, -$800M, Significant Drop | -2%, $10M, Marginal Drop | Expected Value | -100, -$120M, Marginal Drop |
| -5%, -$400M, Marginal Drop | Expected Value | +2%, +$120M, Marginal Increase | Expected Value |
| Expected Value | +3%, +$15M, Marginal Increase | +4%, +$240M, Substantial Increase | +200, +$240M, Marginal Increase |
| +5%, +$400, Marginal Increase | +5%, +$25M, Signficant Increase | +6%, +$360M, Significant Increase | +500, +$600M, Significant Increase |
| +10%, +$800, Significant Increase | | | |

# Alternative Approach: Morphological Analysis

- Generate scenarios by selecting an outcome from each cost driver:

| Labour rates | Material | Engineering design | Artificial Intelligence System |
|---|---|---|---|
| -10%, -$800M, Significant Drop | -2%, $10M, Marginal Drop | Expected Value | -100, -$120M, Marginal Drop |
| -5%, -$400M, Marginal Drop | Expected Value | +2%, +$120M, Marginal Increase | Expected Value |
| Expected Value | +3%, +$15M, Marginal Increase | +4%, +$240M, Substantial Increase | +200, +$240M, Marginal Increase |
| +5%, +$400, Marginal Increase | +5%, +$25M, Signficant Increase | +6%, +$360M, Significant Increase | +500, +$600M, Significant Increase |
| +10%, +$800, Significant Increase | | | |

# Alternative Approach: Morphological Analysis

- Generate scenarios by selecting an outcome from each cost driver:

| Labour rates | Material | Engineering design | Artificial Intelligence System |
|---|---|---|---|
| -10%, -$800M, Significant Drop | -2%, $10M, Marginal Drop | Expected Value | -100, -$120M, Marginal Drop |
| -5%, -$400M, Marginal Drop | Expected Value | +2%, +$120M, Marginal Increase | Expected Value |
| Expected Value | +3%, +$15M, Marginal Increase | +4%, +$240M, Substantial Increase | +200, +$240M, Marginal Increase |
| +5%, +$400, Marginal Increase | +5%, +$25M, Signficant Increase | +6%, +$360M, Significant Increase | +500, +$600M, Significant Increase |
| +10%, +$800, Significant Increase | | | |

# Alternative Approach: Morphological Analysis

- Generate scenarios by selecting an outcome from each cost driver:

| Labour rates | Material | Engineering design | Artificial Intelligence System |
|---|---|---|---|
| -10%, -$800M, Significant Drop | -2%, $10M, Marginal Drop | Expected Value | -100, -$120M, Marginal Drop |
| -5%, -$400M, Marginal Drop | Expected Value | +2%, +$120M, Marginal Increase | Expected Value |
| Expected Value | +3%, +$15M, Marginal Increase | +4%, +$240M, Substantial Increase | +200, +$240M, Marginal Increase |
| +5%, +$400, Marginal Increase | +5%, +$25M, Signficant Increase | +6%, +$360M, Significant Increase | +500, +$600M, Significant Increase |
| +10%, +$800, Significant Increase | | | |

| | | | | |
|---|---|---|---|---|
| **Worst Case ($1.785B)** | +10%, +$800, Significant Increase | +5%, +$25M, Signficant Increase | +6%, +$360M, Significant Increase | +500, +$600M, Significant Increase |

# Alternative Approach: Morphological Analysis

- Common 'brute force' enumeration methodology
- No guarantee of being exhaustive; time-consuming process
- Intuitive, allowing for development of sensible scenarios

| | | | | | |
|---|---|---|---|---|---|
| Scenarios | Worst Case ($1.785B) | +10%, +$800, Significant Increase | +5%, +$25M, Signficant Increase | +6%, +$360M, Significant Increase | +500, +$600M, Significant Increase |
| | Best Case (-$930M) | -10%, -$800M, Significant Drop | -2%, $10M, Marginal Drop | Expected Value | -100, -$120M, Marginal Drop |
| | Construction Cost Inflation (+$425M) | +5%, +$400, Marginal Increase | +5%, +$25M, Signficant Increase | Expected Value | Expected Value |
| | AI System Overage (+$600M) | Expected Value | Expected Value | Expected Value | +500, +$600M, Significant Increase |
| | . | . | . | . | . |
| | . | . | . | . | . |
| | . | . | . | . | . |
| | ED and AI Overage (+$600M) | Expected Value | Expected Value | +6%, +$360M, Significant Increase | +200, +$240M, Marginal Increase |

# Alternative Approach: Simulation-Derived Scenarios

- Application of Monte-Carlo simulation to create scenarios
- Scenarios are comprised of random variates (derived from associated probability distributions) for each cost driver
  - Variables may be discretized for simplicity
  - Distributions may be formed from expert opinion, historical data
- Can generate large quantity of scenarios

| | Labour rates | Material | Engineering design | AI System |
|---|---|---|---|---|
| LowerBound | -10 | -2 | 0 | -1 |
| UpperBound | 10 | 5 | 6 | 5 |
| *1* | 4 | -2 | 5 | 3 |
| *2* | 1 | 4 | 2 | 0 |
| *3* | 9 | -2 | 0 | 0 |
| *4* | -3 | -1 | 3 | 5 |
| *5* | -6 | 5 | 0 | 4 |
| *6* | 0 | 1 | 0 | -1 |
| *7* | 8 | 1 | 3 | 2 |
| *8* | 9 | -1 | 0 | 5 |
| *9* | 10 | -2 | 6 | 5 |
| *10* | -7 | -2 | 0 | 0 |
| *11* | 6 | 3 | 3 | 4 |
| *12* | -6 | 3 | 3 | 4 |
| *13* | -6 | 4 | 4 | 4 |
| *14* | -1 | 5 | 2 | -1 |
| *15* | 5 | 1 | 2 | 1 |
| *…* | 10 | 3 | 1 | 4 |
| *…* | 10 | 0 | 4 | -1 |
| *10000* | 4 | 1 | 0 | 4 |

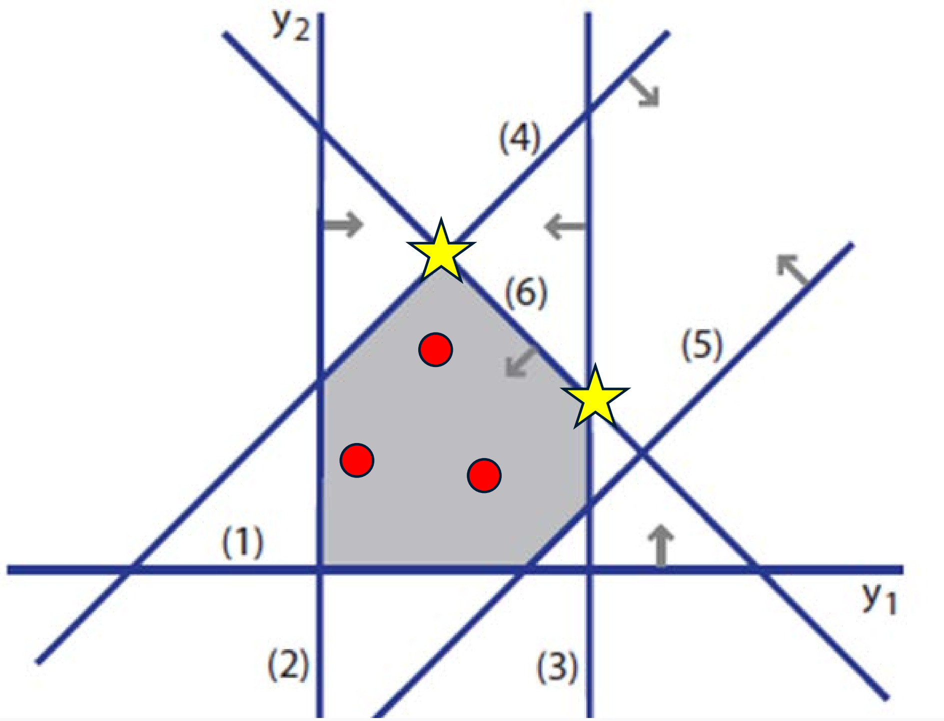
# Alternative Approach: Simulation-Derived Scenarios

- Simple algorithm truncated non-feasible solutions
- Programming could also identify distinctly different solutions
- Characteristics:
  - Computationally easy, can run on common computer; often sufficient for the purposes of explaining results to governance
  - Many unique solutions - difficult to analyze
  - Unlikely to get solutions at extremes – a lot of solutions within the bounds, not at the bounds

# 05. Conclusion

# So What?

- Revelations emerge from the unique combination of cost driver outcomes, within boundaries, ***and at the boundaries***
- Cost risk is often at the margins or at thresholds
- Should be rigor behind tolerance setting
- Enumeration of scenarios is intractable but characterization is possible
- Intuition often won’t lead to certain scenarios (uncommon combos, varying limits)



# Final thoughts on Polytope Analysis

- Pros:
  - High level (suitable for our governance model)
  - Easy to articulate simple relationships between cost drivers
  - Scenarios covered by, or that exceed, specified tolerance level
- Cons:
  - Limited to linear inequality description
  - Only suitable as a top-down analysis, but not bottom-up
  - Only suitable for a limited # of high-level cost risk drivers

# Contact us

bohdan.kaluzny@act.nato.int

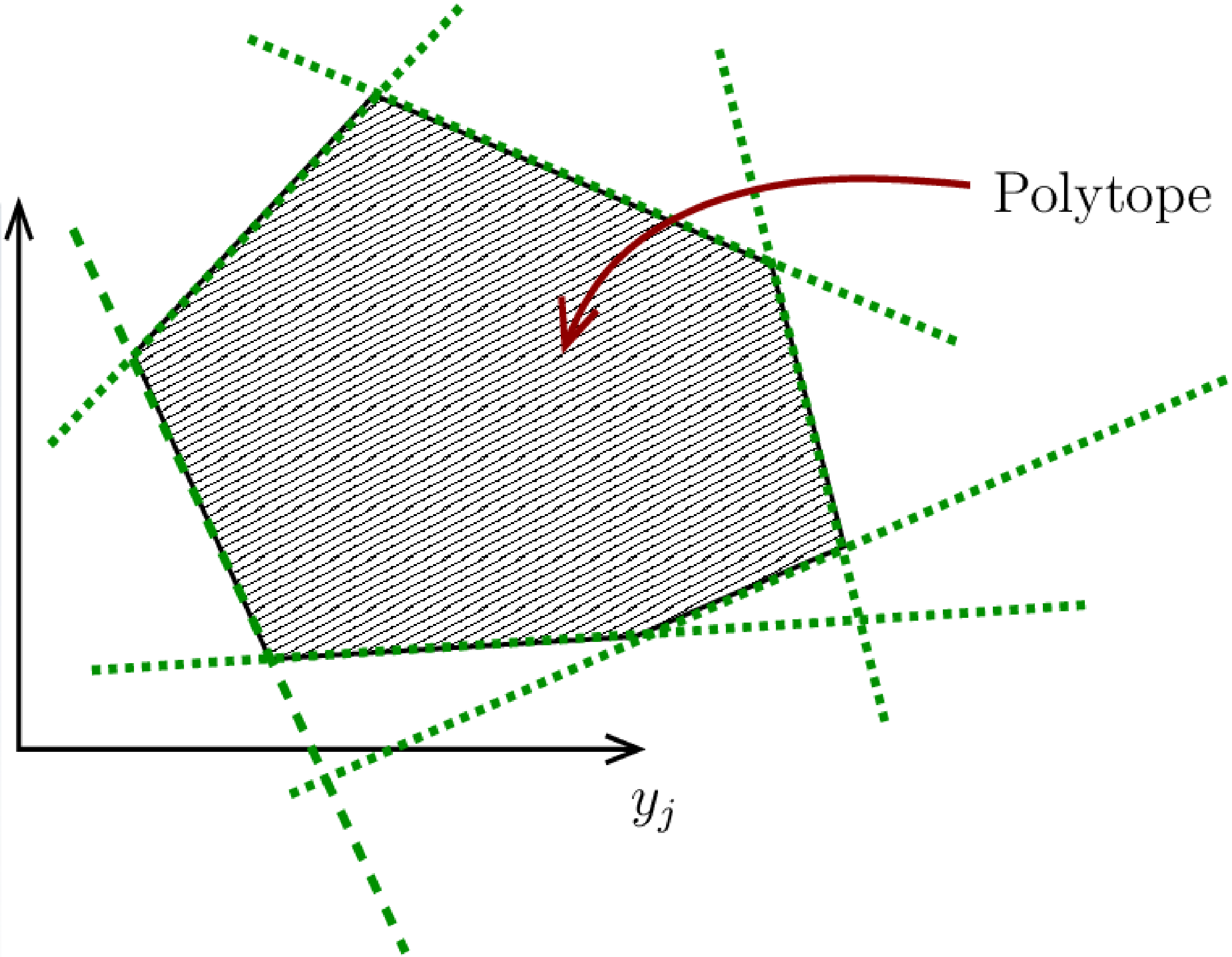
pierre.han@act.nato.int

# 05. Polytopes Explained

# Polytopes

A finite region of n-dimensional space bounded by hyperplanes

(a geometric shape with flat sides, existing in any number of dimensions)

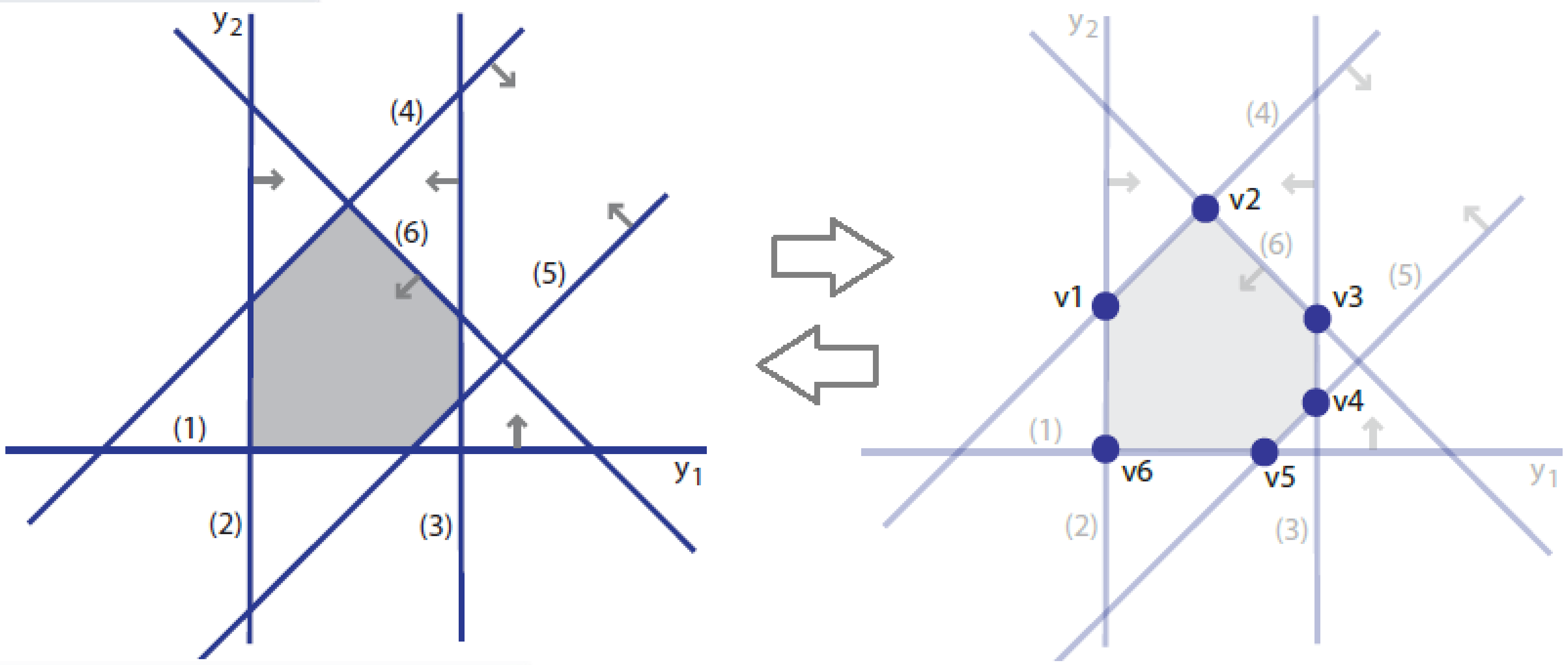


# Main Theorem for Convex Polytopes

P is a bounded intersection of halfspaces/inequalities (H-rep)

if and only if

P is a convex hull of a finite point set (V-rep)



# Main Theorem for Convex Polytopes

P is a bounded intersection of halfspaces/ inequalities (H-rep)

if and only if

P is a convex hull of a finite point set (V-rep)

Representations are mathematically equivalent, but not algorithmically!

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2$$
$$\cdots$$
$$a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nm}x_n \leq b_m$$

↔

Convex Hull of set of points $\{p_1,p_2,\ldots,p_k\}$

each $p_i$ is a n-dimensional point